**Varianta 059**

SUBIECTUL **I**

**a)** $3-4i$ **.**
**b)** $3+4i$
**c)** 0.
**d)** $\frac{9\sqrt{3}}{4}$ **.**
**e)** $\sqrt{2}$
**f)** 1

**SUBIECTUL II**

**1.**
**a)** $27$ **.**
**b)** 8.
**c)** 1.
**d)** $15$.
**e)** 210.

**2.**
**a)** 0.
**b)** $e^x\left(2x+x^2\right)$.
**c)** $3e$.
**d)** $x_0=-2$ este punct de maxim local , iar $x_1=0$ este punct de minim local.
**e)** $e^2-e$.

**SUBIECTUL III**

**a)** $I_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 1-1\\ 0 & 1\end{pmatrix}\in G \quad (a=1)$ **.;**

**b)** Fie $A=\begin{pmatrix}a & 1-a\\ 0 & 1\end{pmatrix}, a\in \mathbf{R}^{\bullet}$**;** det $A=a\neq 0$.

**c)** Fie $A=\begin{pmatrix}x & 1-x\\ 0 & 1\end{pmatrix}, x\in \mathbf{R}^{\bullet}, B=\begin{pmatrix}y & 1-y\\ 0 & 1\end{pmatrix}, y\in \mathbf{R}^{\bullet}$**.**

$A\cdot B=\begin{pmatrix}x & 1-x\\ 0 & 1\end{pmatrix}\begin{pmatrix}y & 1-y\\ 0 & 1\end{pmatrix}=\begin{pmatrix}xy & 1-xy\\ 0 & 1\end{pmatrix}\in G$

**d)** $C=\begin{pmatrix}ab & 1-ab\\ 0 & 1\end{pmatrix}-\begin{pmatrix}ba & 1-ba\\ 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=O_2$ **;**

**e)** $A^2=\begin{pmatrix}a^2 & 1-a^2\\ 0 & 1\end{pmatrix}$; $A^3=A^2\cdot A=\begin{pmatrix}a^3 & 1-a^3\\ 0 & 1\end{pmatrix}$.

**f)** Pentru $n=1$ se verifică uşor.Din **e) ,** avem $A^2 = \begin{pmatrix} a^2 & 1-a^2 \\ 0 & 1 \end{pmatrix}$,deci verificarea este făcută şi pentru $n=2$ .Presupunem $A^k = \begin{pmatrix} a^k & 1-a^k \\ 0 & 1 \end{pmatrix}, \forall k \in \mathbf{N}^\bullet$ şi demonstrăm că $A^{k+1} = \begin{pmatrix} a^{k+1} & 1-a^{k+1} \\ 0 & 1 \end{pmatrix}, \forall k \in \mathbf{N}^\bullet$. $A^{k+1} = A^k \cdot A = \begin{pmatrix} a^k & 1-a^k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix} =$ $\begin{pmatrix} a^{k+1} & 1-a^{k+1} \\ 0 & 1 \end{pmatrix}, \forall k \in \mathbf{N}^\bullet$.Atunci $A^n = \begin{pmatrix} a^n & 1-a^n \\ 0 & 1 \end{pmatrix}, \forall n \in \mathbf{N}^\bullet$.

**g)**Matricea sistemului este $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$. Det $A = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -4 \neq 0$;

sistemul este compatibil unic determinat .Se rezolvă cu regula lui Cramer.
$S = \{(2,1,0)\}$.

SUBIECTUL **IV**

**a)** $f(x) - \frac{1}{x^2} + \frac{1}{(x+1)^2} = 0$ .

**b)** $a_1 = f(1) = \frac{3}{4}; a_2 = f(1) + f(2) = \frac{8}{9}$.

**c)** $\lim\limits_{n\to\infty} f(x) = 0$.Atunci $y=0$ este asimptotă orizontală spre $+\infty$.

**d)** $a_n < a_{n+1}$ devine
$f(1) + f(2) + f(3) + ... + f(n) < f(1) + f(2) + f(3) + ... + f(n) + f(n+1)$.
Obținem $f(n+1) > 0, \forall n \in \mathbf{N}^*$ .Dar $f(n+1) = \frac{2(n+1)}{(n+1)\ (n+2)} > 0\ \forall n \in \mathbf{N}^*$

**e)** Din **a) ,** avem $f(x) - \frac{1}{x^2} + \frac{1}{(x+1)^2}$ =0, $x \in (0,\infty)$,adică $f(x) = \frac{1}{x^2} - \frac{1}{(x+1)^2}$.

$a_n = f(1) + f(2) + f(3) + ... + f(n) = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + ... + \frac{1}{n^2} - \frac{1}{(n+1)^2} =$
1-$\frac{1}{(n+1)^2}, \forall n \in \mathbf{N}^*$.

**f)** $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\left(1 - \frac{1}{(n+1)^2}\right) = 1$.

**g)** $\lim\limits_{n\to\infty} \int_1^n \left( f(x) + \frac{1}{(x+1)^2} \right) dx = \lim\limits_{n\to\infty} \int_1^n \frac{1}{x^2} dx = \lim\limits_{n\to\infty} \left( -\frac{1}{x} \right)\Big|_1^n == \lim\limits_{n\to\infty} \left( -\frac{1}{n} + 1 \right) = 1$.